Esperança de quadrados médios

modelos de parcela subdividida, faixa em DIC e em DBC.

1.1- Modelo 1

$$y_{ijk} = \mu + t_i + b_j + e_{(ij)} + s_k + \delta_{ik} + e_{(ijk)}$$

t_i : efeito do tratamento i da parcela;
b_j : efeito do bloco j
$e_{(ij)}$: erro experimental a_{ij} i, j;
s_k : efeito do tratamento k da subparcela;
δ_{ik} : efeito da interação do trat. i da parcela com o tratamento k da subparcela;
$e_{(ijk)}$: erro experimental entre subparcelas;

FV	GL	QM
bloco	$J-1$	Q_b
TRAT. (T)	$I-1$	Q_T
Ea	$(I-1)(J-1)$	Q_{ea}
T.SUB (S)	$K-1$	Q_s
S x T	$(I-1)(K-1)$	Q_{δ}
Eb	$I(K-1)(J-1)$	Q_{eb}
TOTAL	$IJK-1$	—

Algoritmo de Hicks (1973) MODIFICADO para obtenção dos E (QM).

FIXO OU ALEATÓRIO	I i	J j	K k	$E(QM)$
(F) b_j	I	0	K	$\sigma_b^2 + K\sigma_a^2 + IK\phi_b$
(F) t_i	0	J	K	$\sigma_b^2 + K\sigma_a^2 + IK\phi_t$
(a) $e_{(ij)}$	1	1	K	$\sigma_b^2 + K\sigma_a^2$
(F) S_k	I	J	0	$\sigma_b^2 + IJ\phi_s$
(F) St_{ik}	0	J	0	$\sigma_b^2 + J\phi_{st}$
(a) $e_{(ijk)}$	1	1	1	σ_b^2

1.2 Reparametrizando o modelo para obter o desdobramento da interação: Tratamento da sub-parcela dentro de cada nível do trat. da parcela.

$$y_{ijk} = \mu + b_j + t_i + S_{k(i)} + e_{(ij)} + e_{(ijk)}$$

F/A	FV	I i	J j	K k	QM	$E(QM)$
F	b_j	I	0	K	Q_1	$\sigma_b^2 + K\sigma_a^2 + IK\phi_b$
F	t_i	0	J	K	Q_2	$\sigma_b^2 + K\sigma_a^2 + JK\phi_t$
F	$S_{k(i)}$	1	J	K	Q_3	$\sigma_b^2 + J\phi_{s(t)}$
a	$e_{(ij)}$	1	1	K	Q_4	$\sigma_b^2 + K\sigma_a^2$
a	$e_{(ijk)}$	1	1	1	Q_5	σ_b^2

Trat. parcela dentro da sub-parcela

$$y_{ijk} = \mu + b_j + S_k + t_{i(k)} + e_{(ij)} + e_{(ijk)}$$

F/A	FV	I i	J j	K k	QM	$E(QM)$?
F	b_j	I	0	K	$Q1$	$\sigma_b^2 + K\sigma_a^2 + IK\phi_b$
F	S_k	I	J	0	$Q2$	$\sigma_b^2 + IJ\phi_s + J\phi_{t(s)}$
F	$t_{i(k)}$	1	J	1	$Q3$	$\sigma_b^2 + J\phi_{t(s)}$
a	$e_{(ij)}$	1	1	K	$Q4$	$\sigma_b^2 + K\sigma_a^{2**}$
a	$e_{(ijk)}$	1	1	1	$Q5$	σ_b^2

$\to QME_a \neq QME_a\ (FV_4)$

Variância de comparações de duas médias

5.1- $y_{ijk} = \mu + t_i + b_j + e_{(ij)} + S_{\cdot k} + \delta_{ik} + e_{(ijk)}$

(a) $V(\overline{y}_{i \cdot k} - \overline{y}_{i' \cdot k}) = ?$ { Variância entre duas médias de tratamento da parcela p/ um mesmo nível da sub-parcela

$$\overline{y}_{i\cdot k} = \frac{J\mu}{J} + \frac{J\, t_i}{J} + \sum_j b_j / J + \sum_j e_{ij} / J + S_k + \delta_{ik} + \sum_j e_{ijk} / J$$

$$V\Big(\mu + t_i + \sum_j b_j/J + \sum_j e_{ij}/J + \cancel{S_k} + \delta_{ik} + \frac{\sum_j e_{ijk}}{J} - \mu - t_{i'} - \sum_j b_j/J - \sum_j e_{i'j}/J - \cancel{S_k} - \delta_{i'k} - \frac{\sum e_{i'jk}}{J}\Big) =$$

$$= \boxed{V\Big(t_i - t_{i'} + \frac{\sum e_{ij}}{J} - \frac{\sum e_{i'j}}{J} + \delta_{ik} - \delta_{i'k} + \frac{\sum_j e_{ijk}}{J} - \frac{\sum_j e_{i'jk}}{J}\Big)}$$

E(duplo $\overset{\text{efeitos aleat.}}{\text{produtos}}$) $= \phi$ $\quad V(a) = E(a^2) - E^2(a)$

$E(t_i^2) = t_i^2$; $E(t_{i'})^2 = t_{i'}^2$; $\frac{1}{J^2} E\left(\sum_j e_{ij}\right)^2 = \frac{\sigma_a^2}{J}$

$E\left(\frac{\sum_j e_{i'j}}{J}\right)^2 = \frac{\sigma_a^2}{J}$; $E(\delta_{ik})^2 = \delta_{ik}^2 = E(\delta_{i'k})^2$

$E\left(\frac{\sum_j e_{ijk}}{J}\right)^2 = \frac{\sigma_b^2}{J} = E\left(\frac{\sum e_{i'jk}}{J}\right)$

$E(t_i\, t_{i'}) = t_i \cdot t_{i'}$ $\quad E(t_i\, \delta_{ik}) = t_i\, \delta_{ik}$ $\quad E(t_i\, \delta_{i'k}) = t_i\, \delta_{i'k}$

$E(t_{i'} \cdot \delta_{ik}) = t_{i'}\, \delta_{ik}$ $\quad E(t_{i'}\, \delta_{i'k}) = t_{i'}\, \delta_{i'k}$ $\quad E(\delta_{ik}, \delta_{i'k}) = \delta_{ik}\, \delta_{i'k}$

$E(t_i) = t_i$ $\quad E(t_{i'}) = t_{i'}$ $\quad E(\delta_{ik}) = \delta_{ik}$ $\quad E(\delta_{i'k}) = \delta_{i'k}$

$$E(a^2) = \ell_1^2 + \ell_{1'}^2 + \frac{2\sigma_a^2}{\delta} + \delta_{1k}^2 + \delta_{1'k}^2 + \frac{2\sigma_b^2}{\delta} - 2\ell_1\ell_{1'} +$$
$$+2\ell_1\delta_{1k} - 2\ell_1\delta_{1'k} - 2\ell_{1'}\delta_{1k} +$$
$$+2\ell_{1'}\delta_{1'k} - 2\delta_{1k}\delta_{1'k}$$

$$E(a) = \ell_1 - \ell_{1'} + \delta_{1k} - \delta_{1'k}$$

$$E^2(a) = \ell_1^2 + \ell_{1'}^2 + \delta_{1k}^2 + \delta_{1'k}^2 - 2\ell_1\ell_{1'} + 2\ell_1\delta_{1k} -$$
$$-2\ell_1\delta_{1'k} - 2\ell_{1'}\delta_{1k} + 2\ell_{1'}\delta_{1'k} - 2\delta_{1k}\delta_{1'k}$$

$$\therefore E(a^2) - E^2(a) = \frac{2\sigma_a^2 + 2\sigma_b^2}{\delta}$$

$$V(\bar{y}_{1\cdot k} - \bar{y}_{1'\cdot k}) = \frac{2(\sigma_a^2 + \sigma_b^2)}{\delta}$$

Mas $\hat{\sigma}_a^2 = \dfrac{QMEa - QMEb}{K}$

$\hat{\sigma}_b^2 = QMEb$

$$\Rightarrow \boxed{\begin{aligned}\hat{V}(\bar{y}_{1\cdot k} - \bar{y}_{1'\cdot k}) &= \frac{2}{\delta}\left(\frac{QMEa - QMEb}{K} + QMEb\right)\\ &= \frac{2}{\delta}\left[\frac{QMEa + (K-1)QMEb}{K}\right]\end{aligned}}$$

c.q.m.

δ nº blocos ou de repetições e K níveis do fat. subparcela.

2- Parcela sub-dividida em DIC

$$y_{ijk} = \mu + t_i + e_{j(i)} + S_k + \delta_{ik} + e_{(ijk)}$$

f/a	FV	I i	J j	K k	E(QM)	QM
f	t_i	0	J	K	$\sigma_b^2 + K\sigma_a^2 + JK\phi_t$	Q_t
a	$e_{j(i)}$	1	1	K	$\sigma_b^2 + K\sigma_a^2$	Q_a
f	S_k	I	J	0	$\sigma_b^2 + IJ\phi_S$	Q_S
f	δ_{ik}	0	J	0	$\sigma_b^2 + J\phi_\delta$	Q_δ
a	$e_{(ijk)}$	1	1	1	σ_b^2	Q_b

Variância da diferença de duas médias da parcela para um mesmo nível da subparcela

$$V(\overline{y_{i \cdot k}} - \overline{y_{i' \cdot k}}) = ?$$

$$\overline{y_{i\cdot k}} = \mu + t_i + \frac{\sum_j e_{j(i)}}{J} + S_k + \delta_{i \cdot k} + \frac{\sum_j e_{ijk}}{J}$$

$$V\left[t_i - t_{i'} + \frac{\sum_j e_{j(i')}}{J} - \frac{\sum_j e_{j(i')}}{J} + \delta_{i\cdot k} - \delta_{i'k} + \frac{\sum_j e_{ijk}}{J} - \frac{\sum e_{i'jk}}{J}\right]$$

Expressão idêntica a anterior:

$$\therefore \boxed{\hat{V}(\overline{y_{i\cdot k}} - \overline{y_{i'\cdot k}}) = \frac{2}{J}\left[\frac{QMEa + (k-1)\,QMEb}{K}\right]}$$

GL é obtido por Satterthwaite (1946).

3- Parcela subdividida com fatorial na parcela

$$y_{ijkm} = \mu + b_j + \alpha_i + \beta_k + \delta_{ik} + \ell_{(ijk)} +$$
$$+ \tau_m + \tau\alpha_{im} + \tau\beta_{mk} + \tau\alpha\beta_{ikm} + \ell_{(ijkm)}$$

f/a	FV	I i	J j	K k	M m	QM	E(QM)
f	b_j	I	0	K	M	Q_b	$\sigma_b^2 + M\sigma_a^2 + IKM\phi_b$
f	α_i	0	J	K	M	Q_α	$\sigma_b^2 + M\sigma_a^2 + JKM\phi_\alpha$
f	β_k	I	J	0	M	Q_β	$\sigma_b^2 + M\sigma_a^2 + IJM\phi_\beta$
f	δ_{ik}	0	J	0	M	Q_δ	$\sigma_b^2 + M\sigma_a^2 + JM\phi_\delta$
(a)	$\ell_{(ijk)}$	1	1	1	M	Q_a	$\sigma_b^2 + M\sigma_a^2$
f	τ_m	I	J	K	0	Q_τ	$\sigma_b^2 + IJK\phi_\tau$
f	$\tau\alpha_{im}$	0	J	K	0	$Q_{\tau\alpha}$	$\sigma_b^2 + JK\phi_{\tau\alpha}$
f	$\tau\beta_{mk}$	I	J	0	0	$Q_{\tau\beta}$	$\sigma_b^2 + IJ\phi_{\tau\beta}$
f	$\tau\alpha\beta_{ikm}$	0	J	0	0	$Q_{\tau\alpha\beta}$	$\sigma_b^2 + J\phi_{\tau\alpha\beta}$
(a)	$\ell_{(ijkm)}$	1	1	1	1	Q_b	σ_b^2

(a) Variância entre diferença de 2 tratamentos do fator α dentro de um mesmo nível do fator τ

$$V(\overline{y_{i..m}} - \overline{y_{i'..m}}) = ? \qquad j, k$$

$$\overline{y_{i..m}} = \frac{\sum_j b_j}{J} + \alpha_i + \frac{\sum_k \beta_k}{K} + \frac{\sum_k \delta_{ik}}{K} + \frac{\sum_{jk} \ell_{ijk}}{JK}$$
$$+ \tau_m + \tau\alpha_{im} + \frac{\sum_k \tau\beta_{mk}}{K} + \frac{\sum_k \tau\alpha\beta_{ikm}}{K} +$$
$$+ \frac{\sum \ell_{ijkm}}{JK}$$

$$V\left(\alpha_1 - \alpha_{1'} + \frac{\sum \delta_{1k}}{k} - \frac{\sum \delta_{1'k}}{k} + \frac{\sum e_{1jk}}{\delta k} - \frac{\sum e_{1'jk}}{\delta k} + \right.$$

$$+ T\alpha_{1m} - T\alpha_{1'm} + \frac{\sum_k T\alpha\beta_{1km}}{k} - \frac{\sum_k T\alpha\beta_{1'km}}{k}$$

$$\left. + \frac{\sum e_{1jkm}}{\delta k} - \frac{\sum e_{1'jkm}}{\delta k} \right) =$$

$$= \frac{2\sigma_a^2}{\delta k} + \frac{2\sigma_b^2}{\delta k} = \boxed{\frac{2}{\delta k}(\sigma_a^2 + \sigma_b^2)}$$

$$\boxed{\hat{V}(\overline{y_{1..m}} - \overline{y_{1'..m}}) = \frac{2}{\delta k}\left[\frac{QMEa + (M-1)QMEb}{M}\right]}$$

(b) Variância da diferença de dois trat. do fator β dentro de um mesmo nível de T.

$$\boxed{\hat{V}(\overline{y_{.k.m}} - \overline{y_{.k'.m}}) = \frac{2}{\delta I}\left[\frac{QMEa + (M-1)QMEb}{M}\right]}$$

(c) Variância da ~~~~ dif. de dois níveis do fator α dentro de um mesmo nível de β e T.

$$V(\overline{y_{1k.m}} - \overline{y_{1'k.m}}) = \;?$$

$$\overline{y_{1k.m}} = \mu + \alpha_1 + \beta_k + \delta_{1k} + \frac{\sum e_{1jk}}{\delta} + T_m + T\alpha_{1m} + \alpha T\beta_{mk} + T\alpha\beta_{1km} + \frac{\sum_\delta e_{1jkm}}{\delta}$$

$$V\left(\alpha_1 - \alpha_{1'} + \delta_{1k} - \delta_{1'k} + \frac{\sum_\delta e_{1jk}}{\delta} - \frac{\sum e_{1'jk}}{\delta} + T\alpha_{1m} - T\alpha_{1'm} + T\alpha\beta_{1km} - T\alpha\beta_{1'km} + \frac{\sum e_{1jkm}}{\delta} - \frac{\sum e_{1'jkm}}{\delta} \right) =$$

$$\hat{V}(\overline{y_{i\cdot km}} - \overline{y_{i'\cdot km}}) = \frac{2\sigma_a^2}{J} + \frac{2\sigma_b^2}{J}$$

$$\hat{V}(\overline{y_{i\cdot km}} - \overline{y_{i'\cdot km}}) = \frac{2}{J}\left(\frac{QME_a + (M-1)QME_b}{M}\right)$$

(4) Experimentos em parcelas subdividida no tempo.

$$y_{ijk} = \mu + \alpha_i + b_j + e_{(ij)} + \overset{\text{tempo}}{\beta_k} + e_{(jk)} + \delta_{ik} + e_{(ijk)}$$

F.V.	f/a	I i	J j	K k	QM	E(QM)
α_i	f	0	J	K	Q_α	$\sigma_c^2 + K\sigma_a^2 + JK\phi_\alpha$
b_j	f	I	0	K	Q_b	$\sigma_c^2 + I\sigma_b^2 + K\sigma_a^2 + IK\phi_b$
$e_{(ij)}$	a	1	1	K	Q_a	$\sigma_c^2 + K\sigma_a^2$
β_k	f	I	J	0	Q_β	$\sigma_c^2 + I\sigma_b^2 + IJ\phi_\beta$
$e_{(jk)}$	a	I	1	1	Q_b	$\sigma_c^2 + I\sigma_b^2$
δ_{ik}	f	0	J	0	Q_δ	$\sigma_c^2 + J\phi_\delta$
$e_{(ijk)}$	a	1	1	1	Q_c	σ_c^2

Testador p/ bloco é: $\boxed{QM_{comb.} = Q_b + Q_a - Q_c}$

(a) $V(\overline{\overline{y_{i\cdot\cdot}}} - \overline{y_{i'\cdot\cdot}}) = ?$ $_{j\,e\,k}$

$$\overline{y_{i\cdot\cdot}} = \mu + \alpha_i + \frac{\sum_j b_j}{J} + \frac{\sum_j e_{ij}}{J} + \frac{\sum_k \beta_k}{K} + \frac{\sum_{j,k} e_{jk}}{JK} + \frac{\sum \delta_{ik}}{K} + \frac{\sum e_{ijk}}{JK}$$

$$V(a) = V\left[\alpha_i - \alpha_{i'} + \frac{\sum e_{ij}}{J} - \frac{\sum e_{i'j}}{J} + \frac{\sum_k \delta_{ik}}{K} - \frac{\sum_k \delta_{i'k}}{K} + \frac{\sum e_{ijk}}{JK} - \frac{\sum e_{i'jk}}{JK}\right]$$

$E(a) = ?$ $V(a) = 2J\frac{\sigma_a^2}{J^2} + \frac{2\sigma_c^2}{JK} = \frac{2}{J}\left(\sigma_a^2 + \frac{\sigma_c^2}{K}\right)$

$\epsilon(d_i^2) - \epsilon^2(d_i)$

Mas, $\hat{\sigma}_a^2 = \frac{Q_a - Q_c}{K}$ e $\hat{\sigma}_c^2 = Q_c$

$\therefore \hat{V}(a) = \frac{2}{J}\left(\frac{Q_a - Q_c}{K} + \frac{Q_c}{K}\right) = \boxed{\frac{2}{JK} Q_a}$ c.q.m.

(b) $V(\overline{y_{..k}} - \overline{y_{..k'}}) = V(b) = ?$ i, j

$$\overline{y_{..k}} = \mu + \frac{\Sigma \alpha_i}{I} + \frac{\Sigma b_j}{J} + \frac{\Sigma e_{ij}}{IJ} + \beta_k + \frac{\Sigma_j e_{jk}}{J} + \frac{\Sigma \delta_{ik}}{I} + \frac{\Sigma e_{ijk}}{IJ}$$

$$V(b) = V\left[\beta_k - \beta_{k'} + \frac{\Sigma e_{jk}}{J} - \frac{\Sigma e_{jk'}}{J} + \frac{\Sigma \delta_{ik}}{I} - \frac{\Sigma \delta_{ik'}}{I} + \frac{\Sigma e_{ijk} - \Sigma e_{ijk'}}{IJ}\right]$$

$$= \boxed{\frac{2\sigma_b^2}{J} + \frac{2\sigma_c^2}{IJ}}$$

$$\hat{\sigma}_b^2 = \frac{Q_b - Q_c}{I}$$

$$\therefore \hat{V}(b) = \frac{2}{J}\left[\frac{Q_b - Q_c}{I} + \frac{Q_c}{I}\right] =$$

$$\boxed{\hat{V}(b) = \frac{2}{IJ} Q_b}$$

(c) $V(\overline{y_{i.k}} - \overline{y_{i'.k}}) = ?$ $V(c) = ?$ j

$$\overline{y_{i.k}} = \mu + \alpha_i + \frac{\Sigma b_j}{J} + \frac{\Sigma_j e_{ij}}{J} + \beta_k + \frac{\Sigma_j e_{jk}}{J} + \delta_{ik} + \frac{\Sigma e_{ijk}}{J}$$

$$V(c) = V\left[\alpha_i - \alpha_{i'} + \frac{\Sigma e_{ij} - \Sigma e_{i'j}}{J} + \delta_{ik} - \delta_{i'k} + \frac{\Sigma e_{ijk} - \Sigma e_{i'jk}}{J}\right]$$

$$= \frac{2\sigma_a^2}{J} + \frac{2\sigma_c^2}{J} = \boxed{\frac{2}{J}\left[\sigma_a^2 + \sigma_c^2\right]}$$

$$\hat{V}(c) = \frac{2}{J}\left[\frac{Q_a - Q_c}{k} + \frac{kQ_c}{k}\right]$$

$$\boxed{\hat{V}(c) = \frac{2}{J}\left[\frac{Q_a + (k-1)Q_c}{k}\right]}$$

(d) $V(\overline{y_{1 \cdot k}} - \overline{y_{1 \cdot k'}}) = V(d) = ?$

$$\overline{y_{1 \cdot k}} = \mu + \alpha_1 + \frac{\sum_j b_j}{J} + \frac{\sum e_{1j}}{J} + \beta_k + \frac{\sum e_{ijk}}{J} + \delta_{1k} + \frac{\sum e_{ijk}}{J}$$

$$\therefore V(d) = V\left[\beta_k - \beta_{k'} + \frac{\sum_j e_{jk}}{J} - \frac{\sum e_{jk'}}{J} + \delta_{1k} - \delta_{1k'} + \frac{\sum e_{ijk} - \sum e_{1jk'}}{J}\right]$$

$$= \frac{2\sigma_b^2}{J} + \frac{2\sigma_c^2}{J} = \boxed{\frac{2}{J}\left[\sigma_b^2 + \sigma_c^2\right]}$$

$$\therefore \hat{V}(d) = \frac{2}{J}\left[\frac{Q_b - Q_c}{I} + Q_c\right]$$

$$\boxed{\hat{V}(d) = \frac{2}{J}\left[\frac{Q_b + (I-1)Q_c}{I}\right]}$$

⑤ Parcela sub-dividida no tempo com fatorial na parcela.

$$y_{ijkm} = \mu + R_j + a_i + b_k + \delta_{ik} + e_{(ijk)} +$$

$$+ t_m + e_{(mj)} + ta_{im} + tb_{km} + tab_{ikm} + e_{(ijkm)}$$

a/f	FV	I i	J j	K k	M m	E(QM)
F	R_j	I	0	K	M	$\sigma^2 + IK\sigma_b^2 + M\sigma_a^2 + IKM\phi_R$
F	a_i	0	J	K	M	$\sigma^2 + M\sigma_a^2 + JKM\phi_a$
F	b_k	I	J	0	M	$\sigma^2 + M\sigma_a^2 + IJM\phi_b$
F	δ_{ik}	0	J	0	M	$\sigma^2 + M\sigma_a^2 + JM\phi_\delta$
a	$e_{(ijk)}$	1	1	1	M	$\sigma^2 + M\sigma_a^2$
f	t_m	I	J	K	0	$\sigma^2 + IK\sigma_b^2 + IJK\phi_t$
(a)	$e_{(mj)}$	I	1	K	1	$\sigma^2 + IK\sigma_b^2$
f	ta_{im}	0	J	K	0	$\sigma^2 + JK\phi_{ta}$
f	tb_{km}	I	J	0	0	$\sigma^2 + IJ\phi_{tb}$
f	tab_{ikm}	0	J	0	0	$\sigma^2 + J\phi_{tab}$
a	$e_{(ijkm)}$	1	1	1	1	σ^2

ⓐ Variância da diferença de duas médias do fator A da parcela. $(\bar{Y}_{i\cdot\cdot\cdot})$

$$\bar{Y}_{i\cdots} = \mu + \frac{\sum_j R_j}{J} + a_i + \frac{\sum_k \delta_{ik}}{K} + \frac{\sum_{jk} e_{ijk}}{JK} +$$

$$+ \frac{\sum_m t_m}{M} + \frac{\sum_{mj} e_{mj}}{JM} + \frac{\sum_m ta_{im}}{M} + \frac{\sum tb_{km}}{KM} +$$

$$+ \frac{\sum tab_{ikm}}{KM} + \frac{\sum e_{ijkm}}{JKM}$$

$$V(\bar{Y}_{i\cdots} - \bar{Y}_{i'\cdots}) = V\Big[a_i - a_{i'} + \frac{\sum e_{ijk}}{JK} - \frac{\sum e_{i'jk}}{JK} + \frac{\sum e_{ijkm}}{JKM} - \frac{\sum e_{i'jkm}}{JKM}$$

$$+ \text{efeitos fixos}\Big]$$

$$\boxed{= \frac{2\sigma_a^2}{\delta k} + \frac{2\sigma_e^2}{\delta k M}}$$

$$\hat{V}(\bar{Y}_{1\cdots} - \bar{Y}_{1'\cdots}) = \frac{2}{\delta k} \times \frac{QME_a - QME_c}{M} + \frac{2}{\delta k M} \cdot QME_c$$

$$= \frac{2}{\delta k M}\left[QME_a - QME_c + QME_c \right]$$

$$\boxed{= \frac{2}{\delta k M} QME_a}$$

Da mesma forma:

b) $$\boxed{\hat{V}(\bar{Y}_{\cdot\cdot k\cdot} - \bar{Y}_{\cdot\cdot k'\cdot}) = \frac{2}{\delta M} QME_a}$$

c) $$\boxed{\hat{V}(\bar{Y}_{1\cdot k\cdot} - \bar{Y}_{1'\cdot k\cdot}) = \frac{2}{\delta M} QME_a}$$

d) $$\boxed{\hat{V}(\bar{Y}_{1\cdot k\cdot} - \bar{Y}_{1\cdot k'\cdot}) = \frac{2}{\delta M} QME_a}$$

e) Variância da diferença de duas médias da parcela dentro de um mesmo nível da época (tempo).

$$\bar{Y}_{1\cdots m} = \mu + \frac{\sum R_j}{\delta} + a_i + \frac{\sum_k b_k}{k} + \frac{\sum_k \delta_{ik}}{k} + \frac{\sum_{jk} e_{ijk}}{\delta k} + t_m + \frac{\sum_j l_{jm}}{\delta} + \ldots + \frac{\sum_{jk} l_{ijkm}}{\delta k}$$

$$V(\bar{Y}_{1\cdots m} - \bar{Y}_{1'\cdots m}) = V\left(\frac{\sum e_{ijk}}{\delta k} - \frac{\sum e_{i'jk}}{\delta k} + \frac{\sum l_{jm}}{\delta} - \frac{\sum l_{jm}}{\delta} + \frac{\sum l_{ijkm} - \sum l_{i'jkm}}{\delta k} + \text{efeitos fixos} \right).$$

$$= \frac{2\sigma_a^2}{\delta k} + \frac{2\sigma_c^2}{\delta k} = \frac{2}{\delta k}\left(\sigma_a^2 + \sigma_c^2\right)$$

$$\hat{\sigma}_a^2 = \frac{QME_a - QME_c}{M} \qquad \hat{\sigma}_c^2 = QME_c$$

$$\therefore \hat{V}(\bar{Y}_{1\cdot\cdot\cdot m} - \bar{Y}_{1'\cdot\cdot\cdot m}) = \frac{2}{\delta k M}\left(QME_a - QME_c + MQE_c\right)$$

$$\hat{V}(\bar{Y}_{1\cdot\cdot\cdot m} - \bar{Y}_{1'\cdot\cdot\cdot m}) = \frac{2}{\delta k M}\left[QME_a + (M-1)\,QME_c\right]$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{i\cdot\cdot m} - \bar{Y}_{i'\cdot\cdot m}) = \frac{2}{\delta K}\left[\frac{QME_a + (M-1)\,QME_c}{M}\right]}$$

(f) da mesma forma Variância da diferença de duas médias do fator B de parcela para um mesmo nível de época:

$$\boxed{\hat{V}(\bar{Y}_{\cdot\cdot k m} - \bar{Y}_{\cdot\cdot k' m}) = \frac{2}{I\delta}\left[\frac{QME_a + (M-1)\,QME_c}{M}\right]}$$

(g) Variância p/ desdobramento da diferença de dois níveis do fator A fixando B e T.

$$\bar{Y}_{i\cdot k m} = \mu + \text{efeitos fixos} + \frac{\sum_j e_{ijk}}{\delta} + \frac{\sum_j e_{jm}}{\delta} + \frac{\sum_j e_{ijkm}}{\delta}$$

$$V(d_i) = V\left[\text{efeitos fixos} + \frac{\sum e_{ijk}}{\delta} - \frac{\sum e_{i'jkm}}{\delta} + \frac{\sum e_{jm}}{\delta} - \frac{\sum e_{jm}}{\delta} + \frac{\sum e_{ijkm} - \sum e_{i'jkm}}{\delta}\right]$$

$$= \frac{2\sigma_a^2}{\delta} + \frac{2}{\delta}\sigma_c^2$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{1\cdot k m} - \bar{Y}_{1'\cdot k m}) = \frac{2}{\delta}\left[\frac{QME_a + (M-1)\,QME_c}{M}\right]}$$

(h) Variância p/ dif. de 2 níveis de B fixando A e T

$$\hat{V}(\bar{Y}_{1\cdot k m} - \bar{Y}_{1\cdot k' m}) = \frac{2}{\delta}\left[\frac{QME_a + (M-1)\,QME_c}{M}\right]$$

(i) Variância da diferença das médias de época fixando A.

$$V\left(\text{efeitos fixos} + \cancel{\frac{\sum e_{ijk}}{JK}} - \cancel{\frac{\sum e_{ijk}}{JK}} + \frac{\sum \ell_{jm}}{J} - \frac{\sum \ell_{jm'}}{J} + \frac{\sum e_{ijkm} - \sum e_{ijkm'}}{JK}\right]$$

$$\boxed{= \frac{2\sigma_b^2}{J} + \frac{2\,\sigma_c^2}{JK}}$$

$$\hat{\sigma}_b = \frac{QME_b - QME_c}{IK}$$

$$\therefore \hat{V}(\bar{Y}_{1\cdot\cdot m} - \bar{Y}_{1\cdot\cdot m'}) = \frac{2}{J}\left[\frac{QME_b - QME_c}{IK} + \frac{QME_c}{K}\right]$$

$$\Rightarrow \therefore \boxed{\hat{V}(\bar{Y}_{1\cdot\cdot m} - \bar{Y}_{1\cdot\cdot m'}) = \frac{2}{JK}\left[\frac{QME_b + (I-1)\,QME_c}{I}\right]}$$

(j) Variância da dif. de 2 níveis de época fixando B.

$$\boxed{\hat{V}(\bar{Y}_{\cdot\cdot km} - \bar{Y}_{\cdot\cdot km'}) = \frac{2}{IJ}\left[\frac{QME_b + (K-1)QME_c}{K}\right]}$$

(k) Variância da dif. de 2 níveis de época fixando A e B.

$$V\left(\cancel{\frac{\sum e_{ijk}}{J}} - \cancel{\frac{\sum e_{ijk}}{J}} + \frac{\sum \ell_{jm} - \sum \ell_{jm'}}{J} + \frac{\sum e_{ijkm} - \sum e_{ijkm'}}{J}\right) =$$

$$= \frac{2\sigma_b^2}{J} + \frac{2\sigma_c^2}{J} = \frac{2}{J}\left(\sigma_b^2 + \sigma_c^2\right)$$

$$\therefore \hat{V}(\bar{Y}_{1\cdot km} - \bar{Y}_{1\cdot km'}) = \frac{2}{J}\left(\frac{QME_b - QME_c}{IK} + QME_c\right)$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{1\cdot km} - \bar{Y}_{1\cdot km'}) = \frac{2}{J}\left[\frac{QME_b + (IK-1)\,QME_c}{IK}\right]}$$

~~②~~ ~~[illegible]~~ ① Variância da diferença de duas médias de época (independente de A, B).

$$\bar{Y}_{\cdots m} = \mu + \frac{\sum R_j}{J} + \dots + \frac{\sum_{ij} e_{ij}}{IJK} + \dots + \frac{\sum_j e_{jm}}{J} + \frac{\sum e_{ijkm}}{IJK}$$

$$V(\bar{Y}_{\cdots m} - \bar{Y}_{\cdots m'}) = V\left(\text{efeitos fixos} + \frac{\sum e_{jm} - \sum e_{jm'}}{J} + \frac{\sum e_{ijkm} - \sum e_{ijkm'}}{IJK}\right)$$

$$= \frac{2\sigma_b^2}{J} + \frac{2\sigma_c^2}{IJK} = \frac{2}{J}\left[\sigma_b^2 + \frac{\sigma_c^2}{IK}\right]$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{\cdots m} - \bar{Y}_{\cdots m'}) = \frac{2}{IJK}\, QM\hat{E}_b}$$

⑥ Parcela subdivida no tempo em DIC

é semelhante a parcela subdividida normal → item 2.

7 Parcela subdividida, subdividida no ~~espaço~~ e no tempo

EX. experimento com adubação (A) em DBC com práticas cultivais na sub-parcela, avaliados em vários anos ou épocas distintas.

A → adubos; R – blocos, B – práticas cultivais; C – anos

$$y_{ijkm} = \mu + R_j + a_i + e_{(ij)} + b_k + ab_{ik} + e_{jk(i)} +$$

$$+ c_m + e_{(jm)} + ac_{im} + e_{(ijm)} + bc_{km} + abc_{ikm}$$

$$+ e_{jkm(i)}$$

$e(b) =$ RB + RAB

$e(c) =$ RBC + RABC

J → nº de bloco (3)
K → nº de níveis da cultura (3)
I → nº de níveis do manejo (2)
M → nº de profund. (6)

$E(QM) = ?$

a/F	MODELO	I i	J j	K k	M m	QM	E(QM)
f	r_j	I	0	K	M	Q_r	$\sigma_e^2 + K\sigma_d^2 + IK\sigma_c^2 + M\sigma_b^2 + KM\sigma_a^2 + IKM\phi_r$
f	a_i	0	J	K	M	Q_a	$\sigma_e^2 + K\sigma_d^2 + M\sigma_b^2 + KM\sigma_a^2 + JKM\phi_a$
(a) a	e_{ij}	1	1	K	M	Q_{Ea}	$\sigma_e^2 + K\sigma_d^2 + M\sigma_b^2 + KM\sigma_a^2$
f	b_k	I	J	0	M	Q_b	$\sigma_e^2 + M\sigma_b^2 + IJM\phi_b$
f	ab_{ik}	0	J	0	M	Q_{ab}	$\sigma_e^2 + M\sigma_b^2 + JM\phi_{ab}$
(b) a	$e_{jk(i)}$	1	1	1	M	Q_{Eb}	$\sigma_e^2 + M\sigma_b^2$
f	c_m	I	J	K	0	Q_c	$\sigma_e^2 + K\sigma_d^2 + IK\sigma_c^2 + IJK\phi_c$
(c) a	e_{jm}	I	1	K	1	Q_{Ec}	$\sigma_e^2 + K\sigma_d^2 + IK\sigma_c^2$
f	ac_{im}	0	J	K	0	Q_{ac}	$\sigma_e^2 + K\sigma_d^2 + JK\phi_{AC}$
(d) a	e_{ijm}	1	1	K	1	Q_{Ed}	$\sigma_e^2 + K\sigma_d^2$
f	bc_{km}	I	J	0	0	Q_{bc}	$\sigma_e^2 + IJ\phi_{bc}$
f	abc_{ikm}	0	J	0	0	Q_{abc}	$\sigma_e^2 + J\phi_{abc}$
(e) a	$e_{jkm(i)}$	1	1	1	1	Q_{Ee}	σ_e^2

① Variância da diferença entre duas médias do fator a (só será com efeito fixo)

$$\bar{Y}_{i\cdots} = \frac{\sum_j e_{ij}}{J} + \frac{\sum_{jk} e_{jk(i)}}{JK} + \frac{\sum_{jm} e_{jm}}{JM} + \frac{\sum_{jm} e_{ijm}}{JM} + \frac{\sum e_{jkm(i)}}{JKM}$$

$$V(1) = \frac{2\sigma_a^2}{J} + \frac{2\sigma_b^2}{JK} + \frac{2\sigma_d^2}{JM} + \frac{2\sigma_e^2}{JKM}$$

$$= \frac{2}{J}\left(\sigma_a^2 + \frac{\sigma_b^2}{K} + \frac{\sigma_d^2}{M} + \frac{\sigma_e^2}{KM}\right)$$

$$\boxed{\hat{V}\left(\bar{Y}_{1\cdots} - \bar{Y}_{1'\cdots}\right) = \frac{2}{JKM} QM_{Ea}} \quad \text{(manejo)}$$

J → repetições; I adubos; K práticas culturais; M anos

② Variância entre duas práticas culturais (manejo)

$$\bar{Y}_{\cdot\cdot k\cdot} = \frac{\sum e_{ij}}{IJ} + \frac{\sum e_{jk(i)}}{IJ} + \frac{\sum e_{jm}}{JM} + \frac{\sum e_{ijm}}{IJM} + \frac{\sum e_{jkm(i)}}{IJM}$$

$$V(2) = \frac{2\sigma_b^2}{I\delta} + \frac{2\sigma_e^2}{I\delta M} = \boxed{\frac{2}{I\delta}\left(\sigma_b^2 + \frac{\sigma_e^2}{M}\right)}$$

$$\boxed{\hat{V}(\bar{Y}_{\circ\circ k\circ} - \bar{Y}_{\circ\circ k'\circ}) = \frac{2}{I\delta M} QMEb}$$ cultura

③ Variância da diferença entre dois anos (C).

$$V(3) = ? \quad \bar{Y}_{\circ\circ\circ m} - \bar{Y}_{\circ\circ\circ m'} = \frac{\sum_\delta l_{\delta m}}{\delta} - \frac{\sum l_{\delta m'}}{\delta} + \frac{\sum l_{i\delta m} - \sum l_{i\delta m'}}{I\delta} +$$

$$+ \frac{\sum e_{(i)\delta k m} - \sum e_{(i)\delta k m'}}{I\delta K}$$

$$\therefore V(\bar{Y}_{\circ\circ\circ m} - \bar{Y}_{\circ\circ\circ m'}) = \frac{2\sigma_c^2}{\delta} + \frac{2\sigma_d^2}{I\delta} + \frac{2\sigma_e^2}{I\delta K}$$

$$= \boxed{\frac{2}{\delta}\left(\sigma_c^2 + \frac{\sigma_d^2}{I} + \frac{\sigma_e^2}{IK}\right)}$$

$$\boxed{\hat{V}(\bar{Y}_{\circ\circ\circ m} - \bar{Y}_{\circ\circ\circ m'}) = \frac{2}{I\delta K} QMEc}$$ (profundidades)

④ Variância da diferença de adubações (A) em ~~dois anos~~ fixando práticas culturais (B).

$$\bar{Y}_{i\circ k\circ} = \frac{\sum l_{i\delta}}{\delta} + \frac{\sum l_{\delta k(i)}}{\delta} + \frac{\sum l_{\delta m}}{\delta M} + \frac{\sum l_{i\delta m}}{\delta M} + \frac{\sum l_{\delta m(i)}}{\delta M}$$

$$V(4) = \frac{2\sigma_a^2}{\delta} + \frac{2\sigma_b^2}{\delta} + \frac{2\sigma_d^2}{\delta M} + \frac{2\sigma_e^2}{\delta M}$$ (Manejo dentro de cultura)

$$V(4) = \frac{2}{\delta}\left(\sigma_a^2 + \sigma_b^2 + \frac{\sigma_d^2}{M} + \frac{\sigma_e^2}{M}\right)$$

$$\boxed{\hat{\sigma}_a^2 = \frac{QMEa - QMEb - QMEd + QMEe}{KM}}$$ (1, -1, -1, 1

$$\boxed{\hat{\sigma}_b^2 = \frac{QMEb - QMEe}{M}} \quad \boxed{\hat{\sigma}_d^2 = \frac{QMEd - QMEe}{K}} \quad \boxed{\hat{\sigma}_e^2 = QME}$$

$$\therefore \hat{V}(4) = \frac{2}{\delta}\left[\frac{QME_a - QME_b - QME_d + QME_e}{KM} + \frac{QME_b - QME_e}{M} + \frac{QME_d - QME_e}{KM} + \right.$$

$$\left. + \frac{QME_e}{M}\right]$$

$$= \frac{2}{\delta KM}\left[QME_a - QME_b - \cancel{QME_d} + \cancel{QME_e} + K\,QME_b - \cancel{K\,QME_e} + \cancel{QME_d} - \cancel{QME_e} + \cancel{K\,QME_e}\right]$$

V ©

$$= \frac{2}{\delta KM}\left[QME_a + (K-1)\,QME_b\right]$$

$$\therefore \boxed{\hat{V}(\overline{Y}_{1\cdot k\cdot\cdot} - \overline{Y}_{1'\cdot k\cdot\cdot}) = \frac{2}{\delta M}\left[\frac{QME_a + (K-1)\,QME_b}{K}\right]}$$

⑤ Variância de A fixando um nível de C (anos)

$$\overline{Y}_{1\cdot\cdot m} = \frac{\sum_j e_{1j}}{\delta} + \frac{\sum_{jk} e_{jk(i)}}{\delta K} + \frac{\sum_j e_{jm}}{\delta} + \frac{\sum e_{1jm}}{\delta} + \frac{\sum e_{jkm(i)}}{\delta K}$$

$$V(5) = \frac{2\sigma_a^2}{\delta} + \frac{2\sigma_b^2}{\delta K} + \frac{2\sigma_d^2}{\delta} + \frac{2\sigma_e^2}{\delta K}$$

$$= \boxed{\frac{2}{\delta}\left(\sigma_a^2 + \frac{\sigma_b^2}{K} + \sigma_d^2 + \frac{\sigma_e^2}{K}\right)}$$

$$\therefore \hat{V}(5) = \frac{2}{\delta}\left[\hat{\sigma}_a^2 + \hat{\sigma}_d^2 + \frac{1}{K}\left(\hat{\sigma}_b^2 + \hat{\sigma}_e^2\right)\right]$$

$$= \frac{2}{\delta}\left[\frac{QME_a - QME_b - QME_d + QME_e}{KM} + \frac{QME_d - QME_e}{K} + \right.$$

$$\left. + \frac{1}{K}\left(\frac{QME_b - QME_e}{M}\right) + QME_e\right)\Big]$$

$$= \frac{2}{\delta KM}\left[QME_a - \cancel{QME_b} - QME_d + \cancel{QME_e} + M\,QME_d - \cancel{M\,QME_e} + \cancel{QME_b} - \cancel{QME_e} + \cancel{QME_e}\right.$$

manejo dentro de profund.

$$= \frac{2}{\delta K M}\left[QME a + (M-1)\, QME d \right]$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{i\cdot\cdot m} - \bar{Y}_{i'\cdot\cdot m}) = \frac{2}{\delta K}\left[\frac{QME a + (M-1)\, QME d}{M} \right]}$$

⑥ Variância ~~dos~~ ~~médias~~ para um mesmo nível de A.

$$\boxed{V(\bar{Y}_{i\cdot k\cdot} - \bar{Y}_{i\cdot k'\cdot}) = \frac{2\sigma_b^2}{\delta} + \frac{2\sigma_e^2}{\delta M} = \frac{2}{\delta}\left[\sigma_b^2 + \frac{\sigma_e^2}{M} \right]}$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{i\cdot k\cdot} - \bar{Y}_{i\cdot k'\cdot}) = \frac{2}{\delta M} QME b}$$

cultura dentro de manejo

⑦ Variância da diferença entre 2 níveis de B fixando C

$$\bar{Y}_{\cdot\cdot km} = \frac{\Sigma \ell_{ij}}{I\delta} + \frac{\Sigma e_{jk(i)}}{I\delta} + \frac{\Sigma \ell_{jm}}{\delta} + \frac{\Sigma \ell_{ijm}}{I\delta} + \frac{\Sigma e_{ijkm}}{I\delta}$$

$$\boxed{V(7) = \frac{2\sigma_b^2}{I\delta} + \frac{2\sigma_e^2}{I\delta} = \frac{2}{I\delta}\left[\sigma_b^2 + \sigma_e^2 \right]}$$

$$\therefore \hat{V}(\bar{Y}_{\cdot\cdot km} - \bar{Y}_{\cdot\cdot k'm}) = \frac{2}{I\delta}\left[\frac{QMB - QMEe}{M} + QMEe \right]$$

cult. d profund.

$$\therefore \boxed{\hat{V}(\bar{Y}_{\cdot\cdot km} - \bar{Y}_{\cdot\cdot k'm}) = \frac{2}{I\delta}\left[\frac{QMEb + (M-1)\, QMEe}{M} \right]}$$

⑧ Variância da diferença de 2 níveis de C fixando A.

$$\boxed{V(8) = \frac{2\sigma_c^2}{\delta} + \frac{2\sigma_d^2}{\delta} + \frac{2\sigma_e^2}{\delta K} = \frac{2}{\delta}\left[\sigma_c^2 + \sigma_d^2 + \frac{\sigma_e^2}{K} \right]}$$

$$\therefore \hat{V}(8) = \frac{2}{\delta}\left[\frac{QMEc - QMEd}{IK} + \frac{QMEd - QMEe}{K} + \frac{QMEe}{K} \right]$$

$$= \frac{2}{I\delta K}\left[QME_c - QME_d + I\,QME_d - \cancel{I\,QME_e} + \cancel{I\,QME_e}\right]$$

$$= \frac{2}{I\delta K}\left[QME_c + (I-1)\,QME_d\right]$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{i\cdot\cdot m} - \bar{Y}_{i\cdot\cdot m'}) = \frac{2}{\delta K}\left[\frac{QME_c + (I-1)\,QME_d}{I}\right]}$$

? (9) Variância entre dois níveis de C fixando B.

$$V(9) = \frac{2\sigma_c^2}{\delta} + \frac{2\sigma_d^2}{I\delta} + \frac{2\sigma_e^2}{I\delta} = \boxed{\frac{2}{\delta}\left[\sigma_c^2 + \frac{\sigma_d^2}{I} + \frac{\sigma_e^2}{I}\right]}$$

$$\hat{V}(9) = \frac{2}{\delta}\left[\frac{QME_c - QME_d}{IK} + \frac{QME_d - QME_e}{IK} + \frac{QME_e}{I}\right]$$

$$\hat{V}(9) = \frac{2}{I\delta K}\left[QME_c - \cancel{QME_d} + \cancel{QME_d} - QME_e + K\,QME_e\right]$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{\cdot\cdot km} - \bar{Y}_{\cdot\cdot km'}) = \frac{2}{I\delta}\left[\frac{QME_c + (K-1)\,QME_e}{K}\right]}$$

? (10) Variância entre dois níveis de A fixando B e C

$$\bar{Y}_{i\cdot km} = \text{ef. fixos} + \frac{\sum_\delta e_{i\delta}}{\delta} + \frac{\sum_\delta e_{\delta k(i)}}{\delta} + \frac{\sum_\delta e_{\delta m}}{\delta} + \frac{\sum e_{i\delta m}}{\delta} + \frac{\sum e_{\delta k m(i)}}{\delta}$$

$$V(10) = \frac{2\sigma_a^2}{\delta} + \frac{2\sigma_b^2}{\delta} + \frac{2\sigma_d^2}{\delta} + \frac{2\sigma_e^2}{\delta}$$

$$\boxed{= \frac{2}{\delta}\left(\sigma_a^2 + \sigma_b^2 + \sigma_d^2 + \sigma_e^2\right)}$$

$$\therefore \hat{V}(10) = \frac{2}{\delta}\left[\frac{QME_a - QME_b - QME_d + QME_e}{KM} + \frac{QME_b - QME_e}{M} + \frac{QME_d - QME_e}{K} + QME_e\right]$$

$$= \frac{2}{\delta k M}\left[QME_a - QME_b - QME_d + QME_e + kQME_b - kQME_e + mQME_d - mQME_e + kmQME_e\right]$$

$$= \frac{2}{\delta}\left[\frac{QME_a + (k-1)QME_b + (m-1)QME_d + (km - m - k + 1)QME_e}{kM}\right]$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{h\cdot km} - \bar{Y}_{h'\cdot km}) = \frac{2}{\delta}\left[\frac{QME_a + (k-1)QME_b + (m-1)QME_d + (k-1)(m-1)\,QME_e}{kM}\right]}$$

$$= m(k-1) - (k-1) = (k-1)(m-1)$$

?? (11) Variância entre dois níveis de B fixando A e C.

$$\boxed{V(11) = \frac{2}{\delta}\left[\sigma_b^2 + \sigma_e^2\right]}$$

$$\therefore \hat{V}(11) = \frac{2}{\delta}\left[\frac{QME_b - QME_e}{M} + \frac{M\,QME_e}{M}\right]$$

$$\therefore \boxed{\hat{V}\left[\bar{Y}_{1\cdot km} - \bar{Y}_{1\cdot k'm}\right] = \frac{2}{\delta}\left[\frac{QME_b + (M-1)QME_e}{M}\right]}$$

?? (12) Variância entre dois níveis de C fixando A e B.

$$\boxed{V(12) = \frac{2}{\delta}\left[\sigma_c^2 + \sigma_d^2 + \sigma_e^2\right]}$$

$KI - I$

$I(k-1)$

$$\therefore \hat{V}(12) = \frac{2}{\delta}\left[\frac{QME_c - QME_d}{Ik} + \frac{QME_d - QME_e}{k} + QME_e\right]$$

$$= \frac{2}{\delta}\left[\frac{QME_c + (I-1)QME_d + I(k-1)QME_e}{Ik}\right]$$

$$\therefore \boxed{\hat{V}(\bar{Y}_{1\cdot km} - \bar{Y}_{1\cdot km'}) = \frac{2}{\delta}\left[\frac{QME_c + (I-1)QME_d + I(k-1)QME_e}{Ik}\right]}$$

8) Parcela subdividida no tempo e no espaço em DIC. (ou FAIXA em DIC - subdividido no tempo)

Ex. A → preparo do solo (parcela)
B → Profundidade de avaliação } Sub. parcela - Faixa
C → Época de avaliação

$$y_{ijkm} = \mu + a_i + e_{j(i)} + b_k + ab_{ik} +$$
$$+ e_{jk(i)} + c_m + ac_{im} +$$
$$+ e_{jm(i)} + bc_{km} + abc_{ikm} +$$
$$e_{jkm(i)}$$

1) Variância do dois níveis de A (preparo do solo) fixando B (prof.)

$$V(\bar{Y}_{i.k.} - \bar{Y}_{i'.k.}) = ?\ V(1)$$

$$\frac{\sum_{j,m} Y_{ijkm}}{JM} = \bar{Y}_{i.k.} = \frac{\sum_j e_{j(i)}}{J} + \frac{\sum_j e_{jk(i)}}{J} +$$

$$+ \frac{\sum_j \sum_m e_{jm(i)}}{JM} +$$

$$+ \frac{\sum_j \sum_m e_{jkm(i)}}{JM}$$

$$V(1) = \frac{2\bar{J}}{J^2}\sigma_a^2 + \frac{2}{J}\sigma_b^2 + \frac{2}{JM}\sigma_c^2 + \frac{2}{JM}\sigma_d^2$$

		I	J	K	M		
a/f	modelo	i	j	k	m	QM	E(QM)
f	a_i	0	J	K	M	Q_a	
	$\ell_{j(i)}$	1	1	K	M	$QE_{(a)}$	$\sigma_d^2 + K\sigma_c^2 + M\sigma_b^2 + KM\sigma_a^2$
f	b_k	I	J	0	M	Q_b	
f	ab_{ik}	0	J	0	M	Q_{ab}	
a	$\ell_{jk(i)}$	1	1	1	M	$QE_{(b)}$	$\sigma_d^2 + M\sigma_b^2$
f	c_m	I	J	K	0	Q_c	
f	ac_{im}	0	J	K	0	Q_{ac}	
a	$\ell_{jm(i)}$	1	1	K	1	$QE_{(c)}$	$\sigma_d^2 + K\sigma_c^2$
f	bc_{km}	I	J	0	0	Q_{bc}	$\sigma_d^2 + IJ\phi_{bc}$
f	abc_{ikm}	0	J	0	0	Q_{abc}	$\sigma_d^2 + J\phi_{abc}$
a	$\ell_{jkm(i)}$	1	1	1	1	$QE_{(d)}$	σ_d^2

$$\hat{\sigma}_a^2 = \frac{QME_a - QME_b - QME_c + QME_d}{KM}$$

$$\hat{\sigma}_b^2 = \frac{QME_b - QME_d}{M}$$

$$\hat{\sigma}_c^2 = \frac{QME_c - QME_d}{K}$$

$$\hat{\sigma}_d^2 = QME_d$$

$$\hat{V}(1) = \frac{2}{JKM}\left[QME_a - QME_b - QME_c + QME_d\right] +$$

$$+ \frac{2}{JM}\left[QME_b - QME_d\right] + \frac{2}{JKM}\left[QME_c - QME_d\right] + \frac{2}{JM} QME_d$$

$$\hat{V}(1) = \frac{2}{\delta k m} QME_a + \frac{2(k-1)}{\delta k m} QME_b$$

$$\boxed{\hat{V}(1) = \frac{2}{\delta m}\left[\frac{QME_a + (k-1)\,QME_b}{k}\right]}$$

(2) Variância da dos níveis de A fixando C_0

$$V(2) = V(\overline{Y}_{1..m} - \overline{Y}_{1'.m})$$

$$\overline{Y}_{1..m} = \frac{\sum_j e_{j(i)}}{\delta} + \frac{\sum_j \sum_k e_{jk(i)}}{\delta k} + \frac{\sum_j e_{jm(i)}}{\delta}$$

$$+ \frac{\sum_j \sum_k e_{jkm(i)}}{\delta k}$$

$$V(2) = \frac{2}{\delta}\sigma_a^2 + \frac{2}{\delta k}\sigma_b^2 + \frac{2}{\delta}\sigma_c^2 + \frac{2}{\delta k}\sigma_d^2$$

$$\Rightarrow \hat{V}(2) = \frac{2}{\delta k m}\left[QME_a - \cancel{QME_b} - QME_c + \cancel{QME_d}\right]$$

$$+ \frac{2}{\delta k m}\left[\cancel{QME_b} - \cancel{QME_d}\right] +$$

$$+ \frac{2}{\delta k}\left[QME_c - QME_d\right]$$

$$+ \frac{2}{\delta k} \cancel{QME_d}$$

$$\hat{V}(2) = \frac{2}{\delta k m}\left[QME_a + (m-1)\,QME_c\right]$$

$$\Rightarrow \boxed{\hat{V}(\bar{Y}_{1\cdot\cdot m} - \bar{Y}_{1'\cdot\cdot m}) = \frac{2}{Jk}\left[\frac{QME_a + (M-1)\,QME_c}{M}\right]}$$

(3) Variância p/ ≠ de dois níveis de B fixando-se A.

$$V(\bar{Y}_{1\cdot k\cdot} - \bar{Y}_{1\cdot k'\cdot}) = V(3) = ?$$

$$\bar{Y}_{1\cdot k\cdot} = \frac{\sum_j e_{j(i)}}{J} + \frac{\sum_j e_{jk(i)}}{J} + \frac{\sum_j \sum_m e_{jm(i)}}{JM} + \frac{\sum_j \sum_m e_{jkm(i)}}{JM}$$

$$V(3) = \cancel{\frac{2}{J}\sigma_a^2} \cancel{+ \frac{2}{JM}\sigma_c^2}\ \frac{2}{J}\sigma_b^2 + \frac{2}{JM}\sigma_d^2$$

$$\hat{V}(3) = \cancel{\frac{2}{JkM}\left[QME_a - QME_b - QME_c + QME_d\right]} \cancel{+ \frac{2}{JkM}\left[QME_c - QME_d\right]}$$

$$= \frac{2}{JM}\left[QME_b - \cancel{QME_d}\right] + \cancel{\frac{2}{JM}QME_d}$$

$$\boxed{V(\bar{Y}_{1\cdot k} - \bar{Y}_{1\cdot k'\cdot}) = \frac{2}{JM}\,QME_b}$$

(4) Variância da ≠ de dois níveis de B fixando-se C.

$$V(\bar{y}_{\cdot\cdot km} - \bar{y}_{\cdot\cdot k'm}) = V(4) = ?$$

$$\bar{y}_{\cdot\cdot km} = \frac{\sum_i \sum_j \ell_{j(i)}}{IJ} + \frac{\sum_i \sum_j \ell_{jk(i)}}{IJ} +$$

$$+ \frac{\sum_i \sum_j \ell_{jm(i)}}{IJ} + \frac{\sum_i \sum_j \ell_{jkm(i)}}{IJ}$$

$$\hat{V}(4) = \frac{2}{IJ}\hat{\sigma}_b^2 + \frac{2}{IJ}\hat{\sigma}_d^2$$

$$= \frac{2}{IJM}\left[QME_b - QME_d\right] + \frac{2}{IJ} QME_d$$

$$\boxed{\hat{V}(4) = \frac{2}{IJ}\left[\frac{QME_b + (M-1)\,QME_d}{M}\right]}$$

(5) Variância da ≠ de dois níveis C fixando A.

$$V(\bar{y}_{i\cdot\cdot m} - \bar{y}_{i\cdot\cdot m'}) = V(5) = ?$$

$$V(5) = \frac{2}{J}\sigma_c^2 + \frac{2}{JK}\sigma_d^2$$

$$\hat{V}(5) = \frac{2}{Jk}\left[QME_c - \cancel{QME_d}\right] + \cancel{\frac{2}{Jk} QME_d}$$

$$\boxed{\hat{V}(5) = \frac{2}{Jk} QME_c}$$

6 Variância da ≠ de 2 médias de C fixado B.

$$V(\bar{y}_{\cdot\cdot km} - \bar{y}_{\cdot\cdot km'}) = V(6) = ?$$

$$V(6) = \frac{2}{IJ}\sigma_c^2 + \frac{2}{IJ}\sigma_d^2$$

$$= \frac{2}{IJk}\left[QME_c - QME_d\right] + \frac{2}{IJ} QME_d$$

$$\boxed{\hat{V}(6) = \frac{2}{IJ}\left[\frac{QME_c + (k-1)QME_d}{k}\right]}$$

7 Variância da diferença de 2 médias de A fixado B e C.

$$V(\bar{y}_{i\cdot km} - \bar{y}_{i'\cdot km}) = V(7) = ?$$

$$\bar{y}_{i\cdot km} = \frac{\sum_j e_{j(i)}}{J} + \frac{\sum_j e_{jk(i)}}{J} + \frac{\sum e_{jm(i)}}{J} + \frac{\sum_j e_{jkm(i)}}{J}$$

$$V(7) = \frac{2}{\delta}\sigma_a^2 + \frac{2}{\delta}\sigma_b^2 + \frac{2}{\delta}\sigma_c^2 + \frac{2}{\delta}\sigma_d^2$$

$$\hat{V}(7) = \frac{2}{\delta k m}\left[QME_a - QME_b - QME_c + QME_d\right] + \frac{2}{\delta m}\left[QME_b - QME_d\right] + \frac{2}{\delta k}\left[QME_c - QME_d\right] + \frac{2}{\delta} QME_d$$

$$= \frac{2}{\delta k m} QME_a + \frac{2(k-1)}{\delta k m} QME_b + \frac{2(m-1)}{\delta k m} QME_c + \frac{2(k-1)(m-1)}{\delta k m} QME_d$$

$$\boxed{\hat{V}(7) = \frac{2}{\delta}\left[\frac{QME_a + (k-1)QME_b + (m-1)QME_c + (k-1)(m-1)QME_d}{k m}\right]}$$

(8) Variância da ≠ de dois níveis de B fixados A e C

$$V(\bar{y}_{i\circ km} - \bar{y}_{i\circ k'm}) = V(8) = ?$$

$$V(8) = \frac{2}{\delta}\sigma_b^2 + \frac{2}{\delta}\sigma_d^2$$

$$\hat{V}(8) = \frac{2}{\delta M}\left(QME_b - QME_d\right) + \frac{2}{\delta} QME_d$$

$$\boxed{\hat{V}(8) = \frac{2}{\delta}\left[\frac{QME_b + (M-1)\,QME_d}{M}\right]}$$

(9) Variância da ≠ de dois níveis de C fixando A e B.

$$V(\bar{y}_{1\cdot h m} - \bar{y}_{1 a h_2 m'}) = V(9) = ?$$

$$V(9) = \frac{2}{\delta}\sigma_c^2 + \frac{2}{\delta}\sigma_d^2$$

$$\hat{V}(9) = \frac{2}{\delta K}\left[QME_c - QME_d\right] + \frac{2}{\delta} QME_d$$

$$\boxed{\hat{V}(9) = \frac{2}{\delta}\left[\frac{QME_c + (K-1)\,QME_d}{K}\right]}$$

9c) Modelo de parcela sub-subdividida em DBC

FV		GL	E(QM)	
bl_j (bloco j)		$J-1$		$\hat{\sigma}_a^2 = \frac{QMA-QMB}{KM}$
a_i		$I-1$		
$e_{(1)j}$		$(J-1)(I-1)$ / $1\,1\,K\,M$	$\sigma_c^2 + M\sigma_b^2 + KM\sigma_a^2$	
b_k		$K-1$		
ab_{ik}		$(K-1)(I-1)$		$\hat{\sigma}_b^2 = \frac{QMB-QMC}{M}$
$e_{(1)jk}$		$(J-1)(K-1)I$ / $1\,1\,1\,M$	$\sigma_c^2 + M\sigma_b^2$	
c_m		$M-1$		
ac_{im}		$(M-1)(J-1)$		
bc_{km}		$(M-1)(K-1)$		$\hat{\sigma}_c^2 = QMC$
abc_{ikm}		$(M-1)(I-1)(K-1)$		
$e_{(ijkm)}$		dif. / $1\,1\,1\,1$	σ_c^2	

a) Variância da diferença de duas médias da parcela (A) dentro de um mesmo nível da subparcela B

$$\bar{Y}_{i.k.} = \text{efeitos fixos} + \frac{\sum_j e_{ij}}{J} + \frac{\sum_j e_{(1)jk}}{J} + \frac{\sum_{jm} e_{ijkm}}{JM}$$

$$V(\bar{Y}_{i.k.} - \bar{Y}_{i'.k.}) = \frac{2\sigma_a^2}{J} + \frac{2\sigma_b^2}{J} + \frac{2\sigma_c^2}{JM}$$

$$\hat{V}(\bar{Y}_{i.k.} - \bar{Y}_{i'.k.}) = \frac{2}{J}\left(\frac{QMA-QMB}{KM}\right) + \frac{2}{J}\left(\frac{QMB-QMC}{M}\right) + \frac{2}{J}\left(\frac{QMC}{M}\right)$$

$$\boxed{= \frac{2}{JM}\left[\frac{QMA + (K-1)QMB}{K}\right]}$$

b) Variância da diferença de duas médias da subparcela B dentro de um mesmo nível da parcela A.

$$V(\bar{Y}_{i.k.} - \bar{Y}_{i.k'.}) = \frac{2\sigma_b^2}{J} + \frac{2\sigma_c^2}{JM} =$$

$$\hat{V} = \frac{2}{J}\left(\frac{QMB-QMC}{M}\right) + \frac{2}{J}\left(\frac{QMC}{M}\right) \Rightarrow$$

$$\boxed{\hat{V}(\bar{Y}_{i \cdot k \cdot} - \bar{Y}_{i \cdot k' \cdot}) = \frac{2}{JM} QMB}$$

(c) Variância da diferença de duas médias da parcela (A) fixados os subníveis da sub-sub parcela C.

$$\bar{Y}_{i \cdot \cdot m} = \text{efeitos fixos} + \frac{\sum_j e_{ij}}{J} + \frac{\sum_{jk} e_{ijk}}{JK} + \frac{\sum_{jk} e_{ijkm}}{JK}$$

$$V(\bar{Y}_{i \cdot \cdot m} - \bar{Y}_{i' \cdot \cdot m}) = \frac{2}{J}\sigma_a^2 + \frac{2\sigma_b^2}{JK} + \frac{2\sigma_c^2}{JK}$$

$$\hat{V}(\bar{Y}_{i \cdot \cdot m} - \bar{Y}_{i' \cdot \cdot m}) = \frac{2}{J}\left[\frac{QMA-QMB}{KM} + \frac{QMB-QMC}{KM} + \frac{QMC}{K}\right]$$

$$= \frac{2}{J}\left[\frac{QMA}{KM} + \frac{(M-1)QMC}{KM}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{i \cdot \cdot m} - \bar{Y}_{i' \cdot \cdot m}) = \frac{2}{JK}\left[\frac{QMA + (M-1)QMC}{M}\right]}$$

(d) Variância da diferença de duas médias da sub-sub parcela (C) fixados os níveis da parcela A.

$$V(\bar{Y}_{i \cdot \cdot m} - \bar{Y}_{i \cdot \cdot m'}) = \frac{2\sigma_c^2}{JK}$$

$$\boxed{\hat{V}(\bar{Y}_{i \cdot \cdot m} - \bar{Y}_{i \cdot \cdot m'}) = \frac{2\,QMC}{JK}}$$

(e) Variância da diferença de duas médias da sub-parcela (b) fixados os níveis da sub-sub-parcela (c).

$$\bar{Y}_{..km} = \text{efeitos fixos} + \frac{\sum_{ij} e_{ij}}{I\delta} + \frac{\sum_{ij} e_{ijk}}{I\delta} + \frac{\sum_{ij} e_{ijkm}}{I\delta}$$

$$V(\bar{Y}_{..km} - \bar{Y}_{..k'm}) = \frac{2\sigma_b^2}{I\delta} + \frac{2\sigma_c^2}{I\delta}$$

$$\hat{V} = \frac{2}{I\delta}\left(\frac{QMB - QMC}{M} + QMC\right)$$

$$\boxed{\hat{V}(\bar{Y}_{..km} - \bar{Y}_{..k'm}) = \frac{2}{I\delta}\left[\frac{QMB + (M-1)QMC}{M}\right]}$$

(f) Variância da diferença de duas médias da sub-subparcela (c) fixados os níveis da subparcela (b).

$$V(\bar{Y}_{..km} - \bar{Y}_{..km'}) = \frac{2}{I\delta}\sigma_c^2$$

$$\boxed{\hat{V}(\bar{Y}_{..km} - \bar{Y}_{..km'}) = \frac{2\,QMC}{I\delta}}$$

(g) Variância da diferença de duas médias da parcela (a) fixados os níveis da sub-parcela (b) e da sub-sub-parcela (c).

$$\bar{Y}_{i.km} = \text{efeitos fixos} + \frac{\sum_{j} e_{ij}}{\delta} + \frac{\sum_{j} e_{ijk}}{\delta} + \frac{\sum_{j} e_{ijkm}}{\delta}$$

$$V(\bar{Y}_{i.km} - \bar{Y}_{i'.km}) = \frac{2}{\delta}\sigma_a^2 + \frac{2}{\delta}\sigma_b^2 + \frac{2}{\delta}\sigma_c^2$$

$$\hat{V} = \frac{2}{\delta}\left[\frac{QMA - QMB}{KM} + \frac{QMB - QMC}{M} + QMC\right]$$

$$\hat{V}(\bar{Y}_{1.km} - \bar{Y}_{i'.km}) = \frac{2}{\delta}\left[\frac{QMA}{KM} + \frac{(K-1)\,QMB}{KM} + \frac{(M-1)\,QMC}{M}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{i.km} - \bar{Y}_{i'.km}) = \frac{2}{\delta}\left[\frac{QMA + (k-1)\,QMB + k(M-1)\,QMC}{KM}\right]}$$

(h) Variância da diferença de duas médias de subparcela (b) fixados (a) e (c).

$$V(\bar{Y}_{1.km} - \bar{Y}_{1.k'm}) = \frac{2}{\delta}\sigma_b^2 + \frac{2}{\delta}\sigma_c^2$$

$$\boxed{\hat{V}(\bar{Y}_{1.km} - \bar{Y}_{1.k'm}) = \frac{2}{\delta}\left[\frac{QMB + (M-1)\,QMC}{M}\right]}$$

(i) Variância da diferença de duas médias de sub-subparcela (c) fixados a e b.

$$V(\bar{Y}_{1.km} - \bar{Y}_{i.km'}) = \frac{2}{\delta}\sigma_c^2$$

$$\boxed{\hat{V}(\bar{Y}_{1.km} - \bar{Y}_{i.km'}) = \frac{2}{\delta}\,QMC}$$

(10) Modelo de faixa subdividida no tempo em um DBC (Parcela subdividida no espaço e no tempo em DBC).

FV	i	j	k	m	EQM
bl_j					
p_i					
a $\ell_{(ij)}$	1	1	k	M	$\sigma_g^2 + k\sigma_e^2 + M\sigma_c^2 + kM\sigma_a^2$
f_k					
b ℓ_{jk}	I	1	1	M	$\sigma_g^2 + I\sigma_f^2 + M\sigma_c^2 + IM\sigma_b^2$
pf_{ik}					
c ℓ_{ijk}	1	1	1	M	$\sigma_g^2 + M\sigma_c^2$
t_m					
d ℓ_{jm}	I	1	k	1	$\sigma_g^2 + I\sigma_f^2 + k\sigma_e^2 + Ik\sigma_d^2$
tp_{im}					
e ℓ_{ijm}	1	1	k	1	$\sigma_g^2 + k\sigma_e^2$
tf_{km}					
f ℓ_{jkm}	I	1	1	1	$\sigma_g^2 + I\sigma_f^2$
tpf_{ikm}					
g ℓ_{ijkm}	1	1	1	1	σ_g^2

$$\boxed{\hat{\sigma}_g^2 = QMG} \qquad \boxed{\hat{\sigma}_f^2 = \frac{QMF - QMG}{I}}$$

$$\boxed{\hat{\sigma}_e^2 = \frac{QME - QMG}{k}} \qquad \boxed{\hat{\sigma}_d^2 = \frac{QMD - QME - QMF + QMG}{Ik}}$$

$$\boxed{\hat{\sigma}_c^2 = \frac{QMC - QMG}{M}} \qquad \boxed{\hat{\sigma}_b^2 = \frac{QMB - QMF - QMC + QMG}{IM}}$$

$$\boxed{\hat{\sigma}_a^2 = \frac{QMA - QMC - QME + QMG}{kM}}$$

~~o desdobramento do desdobramento~~

a) Variância para comparar duas médias da parcela (p) fixado ~~[illegible]~~ o nível da faixa (f)

$$\bar{Y}_{1\cdot k\cdot} = \text{efeitos fixos} + \frac{\sum_j e_{ij}}{J} + \frac{\sum_j e_{jk}}{J} + \frac{\sum_j e_{ijk}}{J} +$$

$$+ \frac{\sum_{jm} e_{jm}}{JM} + \frac{\sum_{jm} e_{ijm}}{JM} + \frac{\sum_{jm} e_{jkm}}{JM} +$$

$$+ \frac{\sum_{jm} e_{ijkm}}{JM}$$

$$V(\bar{Y}_{1\cdot k\cdot} - \bar{Y}_{1'\cdot k\cdot}) = \frac{2}{J}\sigma_a^2 + \frac{2\sigma_c^2}{J} + \frac{2\sigma_e^2}{JM} + \frac{2\sigma_f^2}{JM} + \frac{2\sigma_g^2}{JM}$$

$$\hat{V} = \frac{2}{J}\left(\frac{QMA - QMC - QME + QMG}{KM}\right) + \frac{2}{J}\left(\frac{QMC - QMG}{M}\right)$$

$$+ \frac{2}{JM}\left(\frac{QME - QMG}{K}\right) + \cancel{\frac{2}{JM}\left(\frac{QMF - QMG}{I}\right)} + \frac{2}{JM} QMG$$

não aparece (erro)

$$\hat{V} = \frac{2}{JM}\left[\frac{QMA}{K} + \frac{(K-1)QMC}{K}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{1\cdot k\cdot} - \bar{Y}_{1'\cdot k\cdot}) = \frac{2}{JM}\left[\frac{QMA + (K-1)\, QMC}{K}\right]}$$

b) Variância da diferença de duas médias de faixa ~~(f)~~ fixado a mesma parcela (p).

$$V(\bar{Y}_{1\cdot k\cdot} - \bar{Y}_{1\cdot k'\cdot}) = \frac{2}{J}\sigma_b^2 + \frac{2}{J}\sigma_c^2 + \frac{2}{JM}\sigma_f^2 + \frac{2}{JM}\sigma_g^2$$

$$\hat{V} = \frac{2}{JM}\left[\frac{QMB - QMF - QMC + QMG}{I}\right] + \frac{2}{JM}\left[QMC - QMG\right]$$

$$+ \frac{2}{JM}\left[\frac{QMF - QMG}{I}\right] + \frac{2}{JM}\,QMG$$

$$\boxed{\hat{V}(\bar{Y}_{1 \cdot k \cdot} - \bar{Y}_{1 \cdot k' \cdot}) = \frac{2}{JM}\left[\frac{QMB + (I-1)QMC}{I}\right]}$$

(c) Variância da diferença de duas médias da parcela (P_i) fixado o tempo m (t_m).

$$\bar{Y}_{1\cdot\cdot m} = \frac{\sum\limits_{j}^{a} l_{ij}}{J} + \frac{\sum\limits_{jk}^{b} l_{jk}}{JK} + \frac{\sum\limits^{c} l_{ijk}}{JK} + \frac{\sum\limits_{j}^{d} l_{jm}}{J}$$

$$+ \frac{\sum\limits_{j}^{e} l_{ijm}}{J} + \frac{\sum\limits_{jk}^{f} l_{jkm}}{JK} + \frac{\sum\limits_{jk}^{g} l_{ijkm}}{JK}$$

$$V(\bar{Y}_{1\cdot\cdot m} - \bar{Y}_{1'\cdot\cdot m}) = \frac{2}{J}\sigma_a^2 + \frac{2}{JK}\sigma_c^2 + \frac{2}{J}\sigma_e^2 + \frac{2}{JK}\sigma_g^2$$

$$\hat{V} = \frac{2}{JK}\left[\frac{QMA - QMC - QME + QMG}{M}\right] + \frac{2}{JK}\left[\frac{QMC - QMG}{M}\right] +$$

$$+ \frac{2}{JK}\left[QME - QMG\right] + \frac{2}{JK}\,QMG$$

$$\boxed{\hat{V}(\bar{Y}_{1\cdot\cdot m} - \bar{Y}_{1'\cdot\cdot m}) = \frac{2}{JK}\left[\frac{QMA + (M-1)QME}{M}\right]}$$

(d) Variância da diferença de dois tempos fixados a mesma parcela (P_i).

$$V(\bar{Y}_{1\cdot\cdot m} - \bar{Y}_{1\cdot\cdot m'}) = \frac{2}{J}\sigma_d^2 + \frac{2}{J}\sigma_e^2 + \frac{2}{JK}\sigma_f^2 + \frac{2}{JK}\sigma_g^2$$

$$\hat{V} = \frac{2}{\delta k}\left[\frac{QMD - QME - \cancel{QMF + QMG}}{I} + \cancel{QMF - QMG}\right] +$$

$$+ \frac{2}{\delta k}\left[\frac{QMF - QMG}{I} + \cancel{QMG}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{1\cdot\cdot m} - \bar{Y}_{1\cdot\cdot m'}) = \frac{2}{\delta k}\left[\frac{QMD + (I-1)QME}{I}\right]}$$

e) Variância da diferença de duas médias de faixa (f_k) fixado o nível de tempo (t_m).

$$\bar{Y}_{\cdot\cdot km} = \mu + t_k + \frac{\sum_{i,j} e_{ij}}{I\delta} + \frac{\sum_j e_{jk}}{\delta} + \frac{\sum_{i,j} e_{ijk}}{I\delta} + \frac{\sum_j e_{jm}}{\delta} +$$

$$+ \frac{\sum_{i,j} e_{ijm}}{I\delta} + \frac{\sum_j e_{jkm}}{\delta} + \frac{\sum_{i,j} e_{ijkm}}{I\delta}$$

$$V(\bar{Y}_{\cdot\cdot km} - \bar{Y}_{\cdot\cdot k'm}) = \frac{2}{\delta}\sigma_b^2 + \frac{2}{I\delta}\sigma_c^2 + \frac{2}{\delta}\sigma_f^2 + \frac{2}{I\delta}\sigma_g^2$$

$$\hat{V} = \frac{2}{I\delta}\left[\frac{QMB - QMF - \cancel{QMC + QMG}}{M} + \frac{\cancel{QMC - QMG}}{M} + QMF - \cancel{QMG} + \cancel{QMG}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{\cdot\cdot km} - \bar{Y}_{\cdot\cdot k'm}) = \frac{2}{I\delta}\left[\frac{QMB + (M-1)QMF}{M}\right]}$$

f) Variância da diferença de duas médias do tempo (t_m) fixado a mesma faixa (f_k)

$$V(\bar{Y}_{..km} - \bar{Y}_{..km'}) = \frac{2}{J}\sigma_d^2 + \frac{2}{IJ}\sigma_e^2 + \frac{2}{J}\sigma_f^2 + \frac{2}{IJ}\sigma_g^2$$

$$\hat{V} = \frac{2}{IJ}\left[\frac{QMD - \cancel{QME} - QMF + \cancel{QMG}}{k} + \frac{\cancel{QME} - \cancel{QMG}}{k} + QMF - \cancel{QMG} + \cancel{QMG}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{..km} - \bar{Y}_{..km'}) = \frac{2}{IJ}\left[\frac{QMD + (k-1)\,QMF}{k}\right]}$$

g) Variância da diferença entre duas médias da parcela (p_i) fixados t_m e faixa (f_k): $P(T*F)$

$$\bar{Y}_{i.km} = \frac{\sum_j l_{ij}}{J} + \frac{\sum_j l_{jk}}{J} + \frac{\sum_j l_{ijk}}{J} + \frac{\sum_j l_{jm}}{J} + \frac{\sum_j l_{ijm}}{J} + \frac{\sum_j l_{jkm}}{J} + \frac{\sum_j l_{ijkm}}{J}$$

$$V(\bar{Y}_{i.km} - \bar{Y}_{i'.km}) = \frac{2}{J}\sigma_a^2 + \frac{2}{J}\sigma_c^2 + \frac{2}{J}\sigma_e^2 + \frac{2}{J}\sigma_g^2$$

$$\hat{V} = \frac{2}{J}\left[\frac{QMA - QMC - QME + QMG}{KM} + \frac{QMC - QMG}{M} + \frac{QME - QMG}{k} + QMG\right]$$

$$\hat{V} = \frac{2}{J}\left[\frac{QMA - QMC - QME + QMG + kQMC - kQMG + MQME - MQMG + kMQMG}{KM}\right]$$

$$\hat{V} = \frac{2}{J}\left[\frac{QMA + (k-1)\,QMC + (M-1)\,QME + (kM - M - k + 1)\,QMG}{KM}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{i.km} - \bar{Y}_{i'.km}) = \frac{2}{J}\left[\frac{QMA + (k-1)\,QMC + (M-1)\,QME +}{KM} + \frac{(kM - M - k + 1)\,QMG}{KM}\right]}$$

① Variância da diferença de duas médias (fk) fixados pi e tm: F(P*T)

$$V(\bar{Y}_{i \cdot k m} - \bar{Y}_{i \cdot k' m}) = \frac{2}{\delta}(\sigma_b^2 + \sigma_c^2 + \sigma_f^2 + \sigma_g^2)$$

$$\hat{V} = \frac{2}{\delta}\left[\frac{QMB - QMF - QMC + QMG}{IM} + \frac{QMC - QMG}{M} + \frac{QMF - QMG}{I} + QMG\right]$$

$$\hat{V} = \frac{2}{\delta}\left[\frac{QMB + (M-1)QMF + (I-1)QMC + (IM-M-I+1)QMG}{IM}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{i \cdot k m} - \bar{Y}_{i \cdot k' m}) = \frac{2}{\delta}\left[\frac{QMB + (M-1)QMF + (I-1)QMC + (IM-M-I+1)QMG}{IM}\right]}$$

① Variância da diferença de duas médias de tempo fixados parcela (pi) e faixa (fk).

$$V(\bar{Y}_{i \cdot k m} - \bar{Y}_{i \cdot k m'}) = \frac{2}{\delta}(\sigma_d^2 + \sigma_e^2 + \sigma_f^2 + \sigma_g^2)$$

$$\hat{V} = \frac{2}{\delta}\left[\frac{QMD - QME - QMF + QMG}{IK} + \frac{QME - QMG}{K} + \frac{QMF - QMG}{I} + QMG\right]$$

$$\hat{V} = \frac{2}{\delta}\left[\frac{QMD + (I-1)QME + (K-1)QMF + (IK-K-I+1)QMG}{IK}\right]$$

$$\boxed{\hat{V}(\bar{Y}_{i \cdot k m} - \bar{Y}_{i \cdot k m'}) = \frac{2}{\delta}\left[\frac{QMD + (I-1)QME + (K-1)QMF + (IK-K-I+1)QMG}{IK}\right]}$$

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